

Attraction to a radiation-like era in early superstring cosmologies^{*}

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Abstract

Starting from an initial classical four dimensional flat background of the heterotic or type II superstrings, we are able to determine at the string one-loop level the quantum corrections to the effective potential due to the spontaneous breaking of supersymmetry by “geometrical fluxes”. Furthermore, considering a gas of strings at finite temperature, the full “effective thermal potential” is determined, giving rise to an effective non-trivial pressure. The back-reaction of the quantum and thermal corrections to the space-time metric as well as to the moduli fields induces a cosmological evolution that depends on the early time initial conditions and the number of spontaneously broken supersymmetries. We show that for a whole set of initial conditions, the cosmological solutions converge at late times to two qualitatively different trajectories: They are either attracted to (i) a thermal evolution similar to a radiation dominated cosmology, implemented by a coherent motion of some moduli fields, or to (ii) a “Big Crunch” non-thermal cosmological evolution dominated by the non-thermal part of the effective potential or the moduli kinetic energy. During the attraction to the radiation-like era, periods of accelerated cosmology can occur. However, they do not give rise to enough inflation (e -fold $\simeq 0.2$) for the models we consider, where $\mathcal{N} \geq 2$ supersymmetry is spontaneously broken to $\mathcal{N} = 0$.

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1 Introduction

A sensible theoretical description of cosmology has certainly to take into account both the quantum and thermal effects. It is naturally the case for an early phase of the history, close to the Planck time, where quantum fluctuations are of same order of magnitude as the whole size of the universe and where the temperature is extremely high. It has to be the case as well at very late times, where the description of the cosmology should involve the electroweak phase transition and provide the missing link to the quantum particle physics of the standard model (or its supersymmetric stringy extension), account for realistic large scale structure formations, as well as describe correct temperature properties of the cosmic microwave background.

In the framework of strings, the theory of quantum gravity is well defined [1] (at least in certain cases) and can be considered at finite temperature as well [2–4]. Thus, in the stringy framework it is natural to attempt to describe the cosmological evolution of our universe. In the very early times and when the temperature approaches the so called Hagedorn temperature [2–5], we are faced to non-trivial stringy singularities indicating a high temperature non-trivial phase transition. In the literature, there are many speculative proposals concerning the nature of this transition [2, 4, 6–8].

Conceptually, it is quite exotic to understand the early stringy phase utilizing the standard field theoretical geometrical notions [8–11]. Indeed, following for instance Refs [9, 11], the notions of geometry and even topology break down in this very early-time era. Only by considering stringy approaches based on 2-dimensional conformal field theory techniques, there is a hope to analyze further this phase. Actually, some conceptual progress in this direction has been achieved recently [11], but we are not yet in a position to give quantitative descriptions of our stringy universe at these very early times. This stringy-conceptual obstruction however is not an obstacle to the study of the cosmological evolution of our universe for much later times than the Hagedorn transition era, namely for late times such that $t \gg t_H$, [12–14].

A way to bypass the Hagedorn transition ambiguities consists in assuming the emergence of three large space-like directions for $t \gg t_H$, describing the three dimensional space of our universe, and possibly some internal space directions of an intermediate size, characterizing

the scale of the spontaneous breaking of supersymmetry, via geometrical fluxes, or even other (gauge or else) symmetry breaking scales [15]. Within this assumptions, the ambiguities of the “Hagedorn transition exit at t_E ” can be parametrized, for $t \geq t_E$, in terms of initial boundary condition data at t_E . This is precisely the scope of the present work, where we focus on the intermediate cosmological era: $t_E \leq t \leq t_w$, namely, after the “Hagedorn transition exit” and before the electroweak symmetry breaking phase transition at t_w .

Modulo the initial boundary condition data at t_E , the stringy description of the intermediate cosmological era, turns out to be under control, at least for the string backgrounds where supersymmetry is spontaneously broken by “geometrical fluxes” [16, 17]. These fluxes are implemented by a “stringy Scherk-Schwarz mechanism” involving n -internal compactified dimensions which are coupled non-trivially to some of the R-(super-)symmetric charges [18–20] [8, 13, 14]. Furthermore, the finite temperature effects are also implemented by a stringy Scherk-Schwarz compactification attached to the Euclidean time circle of radius $\beta = 1/T$ [2–4, 8, 13, 14]. Here, the R-symmetry charge is just the helicity, so that space-time bosons are periodic while fermions are anti-periodic along the S^1 Euclidean time circle.

The breaking of supersymmetry and the quantum canonical ensemble at temperature T give rise to a non-trivial free energy at the one-loop string level. The latter is evaluated in a large regularized 3-space volume V_3 . This amounts to considering the free energy of a gas of approximately free strings,

$$F = -\frac{\ln Z}{\beta} \simeq -\frac{Z_{1\text{-loop}}^{(\text{string})}}{\beta}, \quad (1.1)$$

where $Z = \text{Tr } e^{-\beta H}$ is the partition function at finite temperature and $Z_{1\text{-loop}}^{(\text{string})}$ is the one-loop connected graph *i.e.* genus one vacuum to vacuum string amplitude computed in the Euclidean background. In these equalities, we use the fact that the free energy of an infinite number of degrees of freedom can be found by a single string loop computation [21] [12–14]. The pressure derived from the free energy, $P = -\partial F / \partial V_3$, has been computed for heterotic and type II models¹. Supersymmetry is spontaneously broken by introducing geometrical fluxes *à la* Scherk-Schwarz involving n internal directions and by finite temperature effects. The pressure acts as a source occurring at 1-loop that backreacts on the original classical space-time metric and moduli fields background. In some cases, the perturbed equations of

¹Type I models realized as orientifolds of type IIB can also be considered. They can be dual to heterotic ones with an initially $\mathcal{N} = 4, 2$ or even 1 supersymmetry [22].

motion for the metric and moduli are shown to admit a cosmological evolution describing a radiation-like dominated era [12–14, 23]. The latter is characterized by a time evolution where the temperature $T(t)$, the supersymmetry breaking scale $M(t)$ and the inverse scale factor $1/a(t)$ remain proportional [12–14, 23]:

$$\textit{Radiation Dominated Solution} : T(t) \propto M(t) \propto 1/a(t) \quad \text{and} \quad H^2 \equiv \left(\frac{\dot{a}}{a}\right)^2 \propto \frac{1}{a^4}. \quad (1.2)$$

Very different time evolutions also exist when temperature effects are neglected [14]. In this case, the supersymmetry breaking scale evolves like:

$$\textit{Moduli Dominated Solution} : M(t) \propto 1/a(t)^{2+4/n} \quad \text{with} \quad T(t) \ll M(t) \quad \text{and} \quad H^2 \propto \frac{1}{a^{8(n+2)/n}}. \quad (1.3)$$

In this work we would like to examine the late time evolution of the universe in the intermediate cosmological era, $t_E \leq t \leq t_w$, in terms of initial boundary conditions (IBC) after the “Hagedorn transition exit at t_E ”. We show that the Radiation Dominated Solution (RDS) and the Moduli Dominated Solution (MDS) obtained for particular IBC are actually generic. For instance, for $n = 1$, there is a whole basin of IBC such that the resulting cosmological evolutions converge – depending on the model – either to the RDS or to the MDS. The first attractor corresponds to an expanding evolution controlled by a Friedmann equation $H^2 \propto 1/a^4$, where H is the Hubble parameter. The second one describes a Big Crunch behavior dominated by the non-thermal radiative corrections. In addition, all models admit a second basin of IBC that yields to another Big Crunch behavior, dominated by the moduli kinetic energy *i.e.* where $H^2 \propto 1/a^6$. Within the scope of our present work, the Big Crunch evolutions can be trusted as long as Hagedorn like singularities are not reached. Our analysis is also limited to the cases where the initial supersymmetry is higher or equal to two, $\mathcal{N} \geq 2$. Although the $\mathcal{N} = 1$ case is quite similar in some respects, it is much more involved in issues like the appearance of an effective cosmological term. It will be analyzed elsewhere.

Restricting to cases with $\mathcal{N} \geq 2$ initial supersymmetry, the attraction to the radiation era depends on the IBC: The convergence to the RDS is either exponential or via damping oscillations. During this process, periods of accelerated expansion can occur. However, they are characterized by an e -fold number of order 0.2, which is too low to account for realistic astrophysical observations. It would be very interesting to extend our analysis to models

with $\mathcal{N} = 1$ initial supersymmetry and see if the e -fold number is larger enough for an inflation era to occur.

In Section 2, the case of a supersymmetry breaking using a single internal direction (*i.e.* $n = 1$) is studied in details. We classify the models and find local basins of attraction. This analysis is completed by a more general exploration of the set of IBC by numerical simulations. In Section 3, the attraction mechanism to the RDS is extended to models where $n = 2$ internal directions participate in the supersymmetry breaking. We determine in Section 4 the e -fold number of the periods of acceleration. Our results, their limitations and our perspectives are summarized in Section 5.

2 Susy breaking involving $n = 1$ internal dimension

In this section, we summarize first the results and notations introduced in Ref. [13] and then we analyze the attractor mechanism. The starting background can be the heterotic, type I or type II superstring compactified on a 6-dimensional space \mathcal{M} that can be either $S^1(R_4) \times T^5$ or $S^1(R_4) \times S^1 \times K3$. The sizes of T^5 and $S^1 \times K3$ are chosen to be of order the Planck length². The supersymmetry breaking is introduced by “geometrical fluxes” via the coupling of the momentum and winding numbers of the Scherk-Schwarz circle $S^1(R_4)$ to an R-symmetry charge [3,13,14,18–20]. The radius of the circle R_4 controls the supersymmetry breaking scale M . In all applications, we will replace $K3$ by its orbifold limit, $K3 \sim T^4/\mathbb{Z}_2$.

In the heterotic and type I cases initially $\mathcal{N} = 4$ or $\mathcal{N} = 2$, the geometrical fluxes break spontaneously all the supersymmetries, $\mathcal{N} \rightarrow 0$. In type II, when the $S^1(R_4)$ coupling involves non-trivially left- and right- moving R-symmetry charges, all initial $\mathcal{N} = 8$, $\mathcal{N} = 4$, or $\mathcal{N} = 2$ supersymmetries are spontaneously broken. When the $S^1(R_4)$ coupling is left-right- “asymmetric” *i.e.* involves non-trivially a left- (or right-) moving R-symmetry charge only, half of the supersymmetries still survive: $\mathcal{N} \rightarrow \mathcal{N}/2$.

²We relax this hypothesis in Ref. [15].

2.1 General setup, gravity-moduli equations

In all cases the computation of the free energy involves for the heterotic or type II superstring the background:

$$S^1(R_0) \times T^3 \times S^1(R_4) \times \mathcal{M}_5 \quad \text{with either } \mathcal{M}_5 = T^5 \text{ or } \mathcal{M}_5 = S^1 \times T^4/\mathbb{Z}_2. \quad (2.1)$$

The T^3 factor stands for the external space with regularized volume V_3 sent to infinity once the thermodynamical limit is taken. $S^1(R_0)$ is the Euclidean time circle of period $\beta = 2\pi R_0$ that defines the inverse temperature (in the string frame). In this direction, the Scherk-Schwarz circle amounts to coupling the momentum and winding numbers of the Euclidean time circle to the space-time fermion number [2–4, 8, 13, 14]. The computation of the free energy density is well defined if the moduli R_0 and R_4 responsible for the spontaneous supersymmetry breaking are larger than the Hagedorn radius R_H , which is of order 1 in string units. At the Hagedorn radius, some winding modes become tachyonic so that the free energy develops a severe IR divergence [2–4, 8, 13, 14]. If our goal is not to describe the phase transition that is occurring at the Hagedorn radius, we can suppose that in the intermediate cosmological era, R_0 and R_4 are large. In Refs [13, 14], it is shown that up to exponentially small terms of order $e^{-\pi R_0}$ or $e^{-\pi R_4}$, the pressure derived from the free energy density takes the following form [12–14, 23],

$$P = T^4 p(z) \quad \text{where} \quad e^z = \frac{M}{T}, \quad (2.2)$$

with the temperature T and supersymmetry breaking scale M defined as,

$$T = \frac{1}{2\pi R_0 \sqrt{\text{Re}S}}, \quad M = \frac{1}{2\pi R_4 \sqrt{\text{Re}S}}. \quad (2.3)$$

In these expressions, the factors involving the dilaton in 4 dimensions, $\text{Re}S = e^{-2\phi_D}$, arise when we work in Einstein frame. In (2.2), p is a linear sum of functions of z with integer coefficients n_T and n_V [13, 14],

$$p(z) = n_T f(z) + n_V \tilde{f}(z), \quad (2.4)$$

where

$$f(z) = \frac{\Gamma(5/2)}{\pi^{5/2}} \sum_{m_0, m_4} \frac{e^{4z}}{[(2m_0 + 1)^2 e^{2z} + (2m_4)^2]^{5/2}}, \quad \tilde{f}(z) = e^{3z} f(-z), \quad (2.5)$$

and the sums are over the integers in \mathbb{Z} . n_T is the number of massless states of the model. If the Scherk-Schwarz circle $S^1(R_4)$ acts non-trivially on the left- and right- moving R-charges, n_V satisfies the inequality:

$$-1 \leq \frac{n_V}{n_T} \leq 1, \quad (2.6)$$

and depends on the choice of R-symmetries used to spontaneously break all the supersymmetries [13, 14]. For an asymmetric $S^1(R_4)$ coupling *i.e.* trivial on the left- (or right-) moving R-symmetry charges, the supersymmetry breaking is partial and thus, there is no contribution \tilde{f} to p . In that case, M is the mass of the $\mathcal{N}/2$ massive gravitini. In the following, we treat both kinds of models keeping in mind that $n_V = 0$ for asymmetric R-symmetry coupling to the momenta and windings of the 4th internal direction.

From a low energy effective field theory point of view, the pressure P is minus the effective potential at finite temperature. It is a source for the quantum fields, namely the dilaton ϕ_D and the modulus R_4 , coupled to gravity [12–14, 23]. It also introduces the “thermal field” R_0 to which there is no associated quantum fluctuation, as can be seen from the string model in light-cone gauge where oscillator modes in the direction 0 are gauged away. The source P is a 1-loop correction that backreacts on the classical 4-dimensional Lorentzian background, via the effective field theory action,

$$S = \int d^4x \sqrt{-g} \left[\frac{1}{2} R - g^{\mu\nu} \left(\partial_\mu \phi_D \partial_\nu \phi_D + \frac{1}{2} \frac{\partial_\mu R_4 \partial_\nu R_4}{R_4^2} \right) + P \right], \quad (2.7)$$

where we write explicitly the fields with non-trivial backgrounds only. Note that computing the 1-loop correction to the kinetic terms is not needed. Indeed, the wave function renormalization of the moduli, (to recast the kinetic terms at 1-loop in a canonical form), would introduce an additional correction to the pressure at second order only. Thus, the full low energy dynamics at first order in string perturbation theory is described by the above action, (2.7).

Before writing the equations of motion, it is useful to redefine the fields,

$$\phi := \sqrt{\frac{2}{3}} (\phi_D - \ln R_4), \quad \phi_\perp := \frac{1}{\sqrt{3}} (2\phi_D + \ln R_4), \quad (2.8)$$

so that the action becomes:

$$S = \int d^4x \sqrt{-g} \left[\frac{1}{2} R - \frac{1}{2} ((\partial\phi)^2 + (\partial\phi_\perp)^2) + P \right], \quad (2.9)$$

where the pressure and the supersymmetry breaking scale M take the form,

$$P = M^4 e^{-4z} p(z), \quad e^z = \frac{M}{T}, \quad M = \frac{e^{\alpha\phi}}{2\pi} \quad \text{with} \quad \alpha = \sqrt{\frac{3}{2}}. \quad (2.10)$$

This shows that in the 2-dimensional moduli space (ϕ_D, R_4) , the effective potential lifts the “no-scale modulus” ϕ , while keeping flat the orthogonal direction ϕ_\perp . Assuming an homogeneous and isotropic cosmological metric with a flat 3-dimensional subspace,

$$ds^2 = -N(t)^2 dt^2 + a(t)^2 (dx_1^2 + dx_2^2 + dx_3^2), \quad (2.11)$$

the Friedmann equation in the $N(t) = 1$ gauge takes the form,

$$3H^2 = \frac{1}{2}\dot{\phi}^2 + \frac{1}{2}\dot{\phi}_\perp^2 + \rho, \quad (2.12)$$

where the energy density ρ is defined by the identity,

$$\rho + P = T \frac{\partial P}{\partial T}, \quad (2.13)$$

and P is understood as a function of the independent variables T and M . As shown in Ref. [14], the expression of ρ follows from the variational principle applied to a gauge choice of the function $N = 1/T$. It is fundamental to stress here that the energy density defined by the variational principle is identical to the one derived from the second law of thermodynamics. From Eqs (2.2) and (2.13), it follows that:

$$\rho = T^4 r(z) \quad \text{where} \quad r = 3p - p_z, \quad (2.14)$$

where z -indices stand for derivatives with respect to $z = \ln(M/T)$. Varying the action with respect to the scale factor a or the scalar fields ϕ and ϕ_\perp , we obtain:

$$2\dot{H} + 3H^2 = -\frac{1}{2}\dot{\phi}^2 - \frac{1}{2}\dot{\phi}_\perp^2 - P, \quad (2.15)$$

$$\ddot{\phi} + 3H\dot{\phi} = \frac{\partial P}{\partial \phi} \equiv \alpha(3P - \rho), \quad (2.16)$$

$$\ddot{\phi}_\perp + 3H\dot{\phi}_\perp = 0 \quad \implies \quad \dot{\phi}_\perp = \sqrt{2} \frac{c_\perp}{a^3}, \quad (2.17)$$

where c_\perp is an arbitrary integration constant. Note that the last identity in Eq. (2.16) is a consequence of the fact that P is a dimension four object, $(M\partial_M + T\partial_T)P \equiv 4P$, and from the definitions (2.10) and (2.13). The conservation of the total energy-momentum tensor follows from the Einstein equations (2.12) and (2.15):

$$\frac{d}{dt} \left(\frac{1}{2}\dot{\phi}^2 + \frac{1}{2}\dot{\phi}_\perp^2 + \rho \right) + 3H \left(\dot{\phi}^2 + \dot{\phi}_\perp^2 + \rho + P \right) = 0. \quad (2.18)$$

Using further the equations of the fields ϕ and ϕ_\perp , we obtain from Eq. (2.18) that the $\{\rho, P\}$ -system is not isolated but is coupled non-trivially to the “no-scale modulus field”, ϕ [24, 25]:

$$\dot{\rho} + 3H(\rho + P) + \alpha\dot{\phi}(3P - \rho) = 0. \quad (2.19)$$

2.2 Classification of the models, effective potential approach

Our aim is to analyze the time behavior of the system described above. For this purpose, one can choose four independent non-linear equations, for instance: Eqs (2.12), (2.16), (2.17) and (2.19), any other choice being equivalent. Furthermore, we find convenient to express the dependence in time in terms of a dependence in the logarithm of scale factor $a(t)$:

$$\lambda := \ln a(t) \quad \implies \quad \dot{y} \equiv \frac{dy}{dt} = H \frac{dy}{d\lambda} \equiv H \overset{\circ}{y}, \quad (2.20)$$

for any function y . In particular, the definition $z = \alpha\phi - \ln(2\pi T)$ implies $\dot{T}/T = H(\alpha\overset{\circ}{\phi} - \overset{\circ}{z})$ that can be used to write

$$\dot{\rho} = HT^4 \left(4r\alpha\overset{\circ}{\phi} + (r_z - 4r)\overset{\circ}{z} \right). \quad (2.21)$$

This equality is useful to express the conservation of energy-momentum (2.19) into a relation giving $\overset{\circ}{\phi}$ in terms of z and $\overset{\circ}{z}$,

$$\alpha\overset{\circ}{\phi} = \mathcal{A}_z(z)\overset{\circ}{z} - 1 \quad \text{where} \quad \mathcal{A}_z(z) = \frac{4r - r_z}{3(r + p)}. \quad (2.22)$$

Integrating, one obtains

$$M = \frac{e^{\mathcal{A}(z)}}{a}, \quad (2.23)$$

where $\mathcal{A}(z)$ involves an integration constant. Using the above relations, the Friedmann Eq. (2.12) becomes

$$3H^2 = \frac{3}{8} H^2 \left((\mathcal{A}_z(z)\overset{\circ}{z} - 1)^2 - 1 \right) + \frac{9}{8} \frac{e^{4(\mathcal{A}(z)-z)}}{a^4} r(z) + \frac{9}{8} \frac{c_\perp^2}{a^6}, \quad (2.24)$$

or,

$$H^2 = T^4 \frac{r(z)}{3 - \mathcal{K}(z, \overset{\circ}{z}, \overset{\circ}{\phi}_\perp)} \quad \text{with} \quad \mathcal{K} = \frac{1}{2\alpha^2} \left(\mathcal{A}_z(z)\overset{\circ}{z} - 1 \right)^2 + \frac{1}{2} \overset{\circ}{\phi}_\perp^2. \quad (2.25)$$

The equations for ϕ and ϕ_\perp , (2.16) and (2.17), take the form,

$$\begin{cases} \frac{r(z)}{3 - \mathcal{K}(z, \overset{\circ}{z}, \overset{\circ}{\phi}_\perp)} \left(\mathcal{A}_z(z) \overset{\circ\circ}{z} + \mathcal{A}_{zz}(z) \overset{\circ}{z}^2 \right) + \frac{r(z) - p(z)}{2} \mathcal{A}_z(z) \overset{\circ}{z} + V_z(z) = 0 \\ \frac{r(z)}{3 - \mathcal{K}(z, \overset{\circ}{z}, \overset{\circ}{\phi}_\perp)} \overset{\circ\circ}{\phi}_\perp + \frac{r(z) - p(z)}{2} \overset{\circ}{\phi}_\perp = 0 \end{cases} \quad (2.26)$$

For the derivation of the above equations, we used the relation $\ddot{y} = \dot{H}\overset{\circ}{y} + H^2\overset{\circ\circ}{y}$ for $y = \phi$ or ϕ_\perp as well as the Eqs (2.22), (2.25) and the linear sum of (2.12) and (2.15). As in Ref. [23], we have introduced the notion of an effective potential $V(z)$, defined by its derivative (and using $\alpha^2 = 3/2$):

$$V_z(z) = r(z) - 4p(z). \quad (2.27)$$

We are going to see in the following section that any solution of the system with constant z , *i.e.* $z \equiv z_c$ where z_c is an extremum of $V(z)$, is giving rise to a particular RDS. The existence of such a critical point depends drastically on the shape of the potential $V(z)$, which is determined by its asymptotic behaviors for $z \rightarrow \pm\infty$. It is obvious from Eq. (2.4) and the definition (2.13) of ρ that $V(z)$ depends only on two parameters, namely, n_T and n_V . The asymptotic behavior $z \rightarrow \pm\infty$ can be easily derived:

$$\bullet \ (n_V/n_T) \neq 0,$$

$$V(z) \underset{z \rightarrow +\infty}{\sim} -e^{4z} \left(\frac{n_V}{n_T} \right) \left(\frac{5}{4} n_T c_5^o \right), \quad \text{with} \quad c_5^o = \frac{\Gamma(5/2)}{\pi^{5/2}} \sum_m \frac{1}{|2m+1|^5}, \quad (2.28)$$

$$\bullet \ (n_V/n_T) \neq -1/15 \equiv - \left(\sum'_m \frac{1}{(2m)^4} \right) / \left(\sum_m \frac{1}{(2m+1)^4} \right),$$

$$V(z) \underset{z \rightarrow -\infty}{\sim} -e^{3z} \left(\frac{n_V}{n_T} + \frac{1}{15} \right) \left(\frac{4}{3} n_T c_4^o \right), \quad \text{with} \quad c_4^o = \frac{\Gamma(5/2)}{\pi^{5/2}} \frac{2}{3} \sum_m \frac{1}{(2m+1)^4}. \quad (2.29)$$

According to the ratio n_V/n_T , three distinct cases can actually arise, (see Fig. 1),

$$\bullet \ \textbf{Case (a)} : \ -1 \leq (n_V/n_T) \leq -1/15$$

$V(z)$ increases monotonically from 0 to $+\infty$.

$$\bullet \ \textbf{Case (b)} : \ -1/15 < (n_V/n_T) < 0$$

$V(z)$ admits a unique minimum z_c , with $p(z_c)$ positive. When $(n_V/n_T) \rightarrow 0_-$ the critical value z_c goes to $+\infty$; when $(n_V/n_T) \rightarrow (-1/15)_+$ the critical value $z_c \rightarrow -\infty$.

- **Case (c) :** $0 \leq (n_V/n_T) \leq 1$

$V(z)$ decreases monotonically from 0 to $-\infty$.

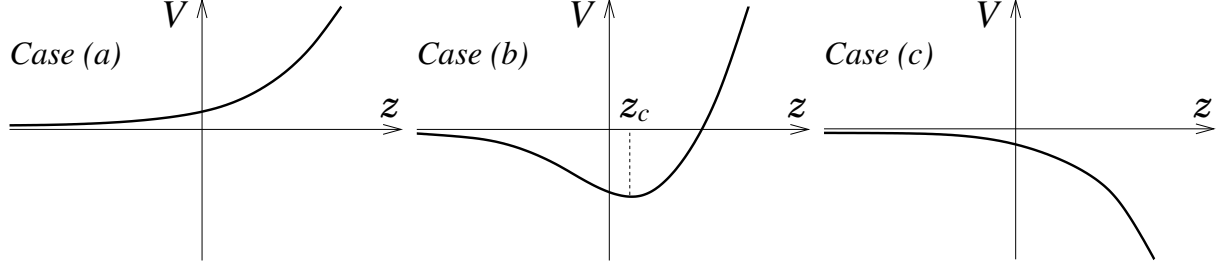


Figure 1: The shape of the potential $V(z)$ in Eq. (2.26) depends on the ratio n_V/n_T . The Cases (a), (b) and (c) correspond to $-1 \leq n_V/n_T \leq -1/15$, $-1/15 < n_V/n_T < 0$ and $0 \leq n_V/n_T \leq 1$, respectively.

Qualitatively, one can expect that the system represented by z can slide along its potential. It may run away in Case (a), $z(t) \rightarrow -\infty$, be stabilized in Case (b), $z(t) \rightarrow z_c$, and run away in Case (c), $z(t) \rightarrow +\infty$. Moreover, these behaviors should imply new ones by time reversal. In particular, other run away behaviors are expected: $z(t) \rightarrow +\infty$ in Case (a), $z(t) \rightarrow \pm\infty$ in Case (b), and $z(t) \rightarrow -\infty$ in Case (c). However, the study of the run away $z \rightarrow -\infty$ is out of the scope of the present work. This is due to the fact that this limit amounts to $R_4 \gg R_0 > 1$, so that the thermal system should be studied in 5 dimensions. In the following, we will only consider the other behaviors, namely the stabilization of z and the run away $z \rightarrow +\infty$. Since the latter amounts to $R_0 \gg R_4 > 1$, it is consistent with our analysis in four dimensions but with negligible thermal effects.

2.3 Local attraction to the RDS

Clearly, the first equation in (2.26) admits a constant z solution whenever V has an extremum, *i.e.* in Case (b). It follows from Eq. (2.23) that M , T and $1/a$ are propotionnal during the evolution,

$$M(t) = e^{z_c} T(t) = \frac{e^{A(z_c)}}{a(t)}. \quad (2.30)$$

The time dependence of the scale factor is dictated by the Friedmann Eq. (2.24) that simplifies to:

$$3H^2 = \frac{c_r^2}{a^4} + \frac{c_m^2}{a^6} \quad \text{where} \quad c_r^2 = \frac{9}{2} e^{4(A(z_c)-z_c)} p(z_c), \quad c_m^2 = \frac{9}{8} c_\perp^2. \quad (2.31)$$

Integrating, t can be expressed in terms of the scale factor,

$$t(a) = t_0 \int_0^{a/a_0} \frac{x^2 dx}{\sqrt{1+x^2}}, \quad t_0 = \sqrt{3} \frac{c_m^2}{|c_r|^3}, \quad a_0 = \left| \frac{c_m}{c_r} \right|, \quad \forall a \geq 0, \quad (2.32)$$

up to an overall sign. There are thus two monotonic solutions mapped to one another by time reversal. For the expanding one, the universe life starts in a “moduli kinetic energy dominated era” characterized by a total energy density $\simeq c_m^2/a^6$, followed by a “radiation dominated era” with total energy density $\simeq c_r^2/a^4$. For large enough times, the evolution always enters in the radiation era: It is asymptotic to the critical solution $z \equiv z_c$, $c_m = 0$ *i.e.* (2.30) where

$$a(t) = \sqrt{t} \times \left(\frac{4}{3} c_r^2 \right)^{1/4}, \quad \dot{\phi}_\perp = 0. \quad (2.33)$$

To study the stability of this solution, we analyze its behavior under small perturbations. In terms of the variable $\lambda = \ln a$, we define,

$$z(\lambda) = z_c + \varepsilon(\lambda), \quad \overset{\circ}{\phi}_\perp(\lambda) = \overset{\circ}{\varepsilon}_\perp(\lambda), \quad (2.34)$$

for small ε and $\overset{\circ}{\varepsilon}_\perp$. At first order, the system (2.26) becomes

$$\begin{cases} \overset{\circ}{\varepsilon} + \overset{\circ}{\varepsilon} + \xi \varepsilon = 0 & \text{where} \quad \xi = 10 \left. \frac{p - p_{zz}}{19p + p_{zz}} \right|_{z_c}, \\ \overset{\circ}{\varepsilon}_\perp + \overset{\circ}{\varepsilon}_\perp = 0. \end{cases} \quad (2.35)$$

The coefficient ξ is actually a function of the model dependent parameter n_V/n_T , since the latter appears explicitly in the definition of p and implicitly via z_c . Numerically, one finds that ξ is always positive. It increases from 0 to $+\infty$ when n_V/n_T varies from $-1/15$ to 0. The solution of the system (2.35) is

$$\overset{\circ}{\varepsilon}_\perp(\lambda) = c_3 e^{-\lambda}, \quad \varepsilon(\lambda) = \begin{cases} c_1 e^{-\lambda(1+\sqrt{1-4\xi})/2} + c_2 e^{-\lambda(1-\sqrt{1-4\xi})/2} & \text{if } 0 < \xi < 1/4 \\ e^{-\lambda/2}(c_1 + c_2 \lambda) & \text{if } \xi = 1/4 \\ e^{-\lambda/2} \left(c_1 \cos \left(\lambda \sqrt{\xi - 1/4} \right) + c_2 \sin \left(\lambda \sqrt{\xi - 1/4} \right) \right) & \text{if } \xi > 1/4 \end{cases} \quad (2.36)$$

where $c_{1,2,3}$ are integration constants determined by the IBC. Since $\varepsilon(\lambda)$ and $\overset{\circ}{\varepsilon}_\perp(\lambda)$ converge to 0 for large λ in all cases, the critical solution is stable under small perturbations. In other words, the solution arising for arbitrary IBC such that $\varepsilon(0)$, $\overset{\circ}{\varepsilon}(0)$ and $\overset{\circ}{\varepsilon}_\perp(0)$ are small is attracted toward the critical one, $z \equiv z_c$, $\dot{\phi}_\perp \equiv 0$. The behavior of $\varepsilon(\lambda)$ is oscillating with damping when $0 < \xi < 1/4$, and exponentially convergent when $\xi \geq 1/4$. The special value $\xi = 1/4$ corresponds to $n_V/n_T \simeq -0.0246$.

2.4 Global attraction to the RDS

Since we have shown the existence of a local basin of attraction around the critical solution given in equations (2.30) and (2.33), we want to know if this property is valid for more generic IBC. Actually, any initial values $(z_0, \overset{\circ}{z}_0, \overset{\circ}{\phi}_{\perp 0})$ at $\lambda = 0$ are allowed, as long as the positivity of the right hand side of the Friedmann Eq. (2.25) is guaranteed³, *i.e.*

$$\frac{r(z_0)}{3 - \mathcal{K}(z_0, \overset{\circ}{z}_0, \overset{\circ}{\phi}_{\perp 0})} \geq 0. \quad (2.37)$$

We have studied numerically the system (2.26) for generic IBC satisfying the previous bound. Implementing in the code a positive increment for $\lambda = \ln a$ simulates the phases of the evolutions where the scale factor increases. We observe that the right hand side of (2.25) never changes of sign. This means that the scale factor $a(t)$ is monotonic⁴. We always find that the solutions satisfy $z(\lambda) \rightarrow z_c$ and $\overset{\circ}{\phi}_{\perp}(\lambda) \rightarrow 0$ as $\lambda \rightarrow +\infty$, after possible oscillations. To illustrate the convergent behavior for large oscillations, we present a numerical example on Fig. 2. It is obtained for $n_V/n_T = -0.02$ that corresponds to $z_c \simeq 0.272$, with IC $(z_0, \overset{\circ}{z}_0, \overset{\circ}{\phi}_{\perp 0}) = (0.4, 0.8, 0.5)$. We conclude that for generic IBC, the expanding evolutions

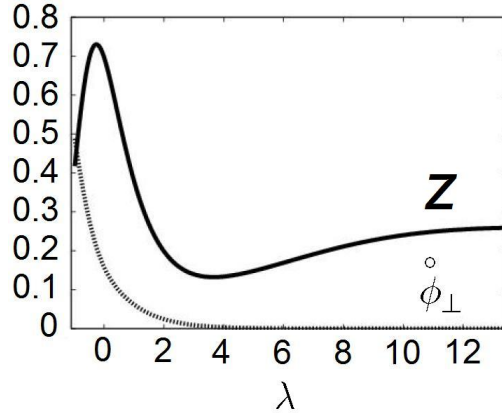


Figure 2: *Examples of damping oscillations of $z(\lambda)$ (solid curve) and convergence to zero of $\overset{\circ}{\phi}_{\perp}(\lambda)$ (dotted curve) illustrating the dynamical attraction toward the critical solution $z \equiv z_c \simeq 0.272$, $\dot{\phi}_{\perp} \equiv 0$, obtained for $n_V/n_T = -0.02$. The initial conditions are $(z_0, \overset{\circ}{z}_0, \overset{\circ}{\phi}_{\perp 0}) = (0.4, 0.8, 0.5)$.*

are attracted by the RDS that satisfies $z \equiv z_c$, $\dot{\phi}_{\perp} \equiv 0$ *i.e.* $3H^2 = c_r^2/a^4$ in Eq. (2.30).

³IBC such that the right hand side of (2.25) is negative correspond to solutions in Euclidean time.

⁴In general, a change of sign in the Friedmann equation during a simulation would be a numerical artifact meaning that H actually vanishes when the scale factor reaches an extremum.

The local behavior (2.36) also shows that the trajectory $z \equiv z_c, \dot{\phi}_\perp \equiv 0$ is repulsive, when λ decreases *i.e.* for evolutions where the scale factor is contracting. The Big Crunch solution obtained by time reversal on (2.33) is thus only formal, since it is highly unstable under small fluctuations. What we observe by numerical simulations is that for generic IBC, when we choose a negative increment for λ , then either $z \rightarrow +\infty$, or z oscillates with a greater and greater amplitude. To understand better the dynamics of the contracting evolutions, we are thus led to analyze carefully the system in the regime $z \gg 1$.

2.5 Attraction to Big Crunch eras

We have seen that when the potential (2.27) admits a minimum z_c (*Case (b)*), the expanding cosmological evolutions for arbitrary IBC satisfy $z(t) \rightarrow z_c$. The shape of the potential in *Case (c)* of Fig. 1 suggests that when $n_V \geq 0$, the dynamics could admit a run away behavior $z(t) \rightarrow +\infty$. We also noticed at the end of the previous subsection that the contracting evolutions in *Case (b)* involve the large and positive value regime of z . For these reasons, we need to study the system for $z \gg 1$. In this limit the thermal effects $\mathcal{O}(T^4)$ are sub-dominant compare to the supersymmetry breaking effects $\mathcal{O}(M^4)$. The expansion of $p(z)$ contains two monomials in e^z ,

$$p(z) = n_V e^{4z} c_5^o + (n_T c_4^o + n_V c_4^e) + \dots \quad \text{where} \quad c_4^e = \frac{\Gamma(5/2)}{\pi^{5/2}} \frac{2}{3} \sum'_m \frac{1}{(2m)^4}, \quad (2.38)$$

and c_5^o, c_4^o are defined in Eqs (2.28) and (2.29). The dots stand for exponentially suppressed terms $\mathcal{O}(e^{-e^z})$. It follows that $\mathcal{A}_z(z) \simeq 1$ and the conservation of the energy-momentum Eq. (2.22) becomes

$$\dot{z} = \dot{M}/M + 1 \quad \implies \quad \frac{\dot{T}}{T} = -H \quad i.e. \quad aT = a_0 T_0, \quad (2.39)$$

where $a_0 T_0$ is a positive constant depending on the IBC. From the expansion of $p(z)$ for large z in Eq. (2.38), it is clear that the behavior of the system is drastically different when $n_V \neq 0$ and when $n_V = 0$.

• $n_V \neq 0, z \gg 1$:

We have $\rho \simeq -P \simeq -M^4 n_V c_5^o$. The Friedmann Eq. (2.12) and the linear sum of Eqs (2.12)

and (2.15), together with the matter field Eqs (2.16) and (2.17), become

$$3H^2 = \frac{1}{2}\dot{\phi}^2 + \frac{1}{2}\dot{\phi}_\perp^2 - M^4 n_V c_5^o, \quad (2.40)$$

$$\dot{H} + 3H^2 = -M^4 n_V c_5^o, \quad (2.41)$$

$$\alpha\ddot{\phi} + 3H\alpha\dot{\phi} = 4\alpha^2 M^4 n_V c_5^o, \quad \ddot{\phi}_\perp + 3H\dot{\phi}_\perp = 0. \quad (2.42)$$

The above equations would also arise if we had considered the system at zero temperature. This is due to the fact that when temperature effects are not taken into account, the sources of Einstein gravity always satisfy $\rho = -P$ equal to minus the effective potential (see Eq. (2.13)). However, this does not mean that in our case we shall find that $T(t) \rightarrow 0$. Actually, the regime $z \gg 1$ *i.e.* $M \gg T$ corresponds to thermal effects screened by radiative corrections, even if the temperature can be large. Combining Eq. (2.41) and Eq. (2.42) one determines the scalar fields in terms of the scale factor, (with $\alpha = \sqrt{3/2}$),

$$\alpha\dot{\phi} = -4\alpha^2 H + \frac{c_\phi}{a^3} \quad \Longrightarrow \quad M = \frac{e^{c_\phi \int^t \frac{dt'}{a(t')^3}}}{a^6} \quad \text{and} \quad \dot{\phi}_\perp = \sqrt{2} \frac{c_\perp}{a^3}, \quad (2.43)$$

where c_ϕ and c_\perp are constants depending on the IBC. The Friedmann Eq. (2.40) takes then the form

$$3H^2 = \frac{1}{3} \left(6H - \frac{c_\phi}{a^3} \right)^2 + \frac{c_\perp^2}{a^6} - M^4 n_V c_5^o. \quad (2.44)$$

We will consider separately the cases $c_\phi = 0$ and $c_\phi \neq 0$.

(i) $c_\phi = 0$:

$$M = M_0 \left(\frac{a_0}{a} \right)^6, \quad 3H^2 = -\frac{c_m^2}{a^6} + \frac{c_M^2}{a^{24}}, \quad \text{with} \quad c_m = \frac{c_\perp^2}{3}, \quad c_M^2 = \frac{n_V}{3} M_0^4 c_5^o a_0^{24}, \quad (2.45)$$

where $M_0 a_0^6$ is a positive constant. This solution exists only for $n_V > 0$. If we also have $c_\perp = 0$, up to time reversal, the evolution describes a Big Crunch occurring at a finite time t_{BC} ,

$$a(t) = a_0 (t_{\text{BC}} - t)^{1/12} \times (16 M_0^4 n_V c_5^o)^{1/24}, \quad t < t_{\text{BC}}. \quad (2.46)$$

Since $a \sim 1/T$, it follows that $e^z \propto aM \propto 1/a^5$, which is large if $t \lesssim t_{\text{BC}}$ and implies a run away of z , consistently with our hypothesis $z \gg 1$. However, close to the Big Crunch, the scale M (as well as T , even if $T \ll M$) is formally diverging. This implies that the present analysis has to be restricted to the regime where $M < M_H$ *i.e.* $z < (5/6) \ln M_H$, where M_H is the supersymmetry breaking scale above which a Hagedorn-like transition occurs. To bypass this limit, one could try to extend our study to Hagedorn singularity free models [8].

For $c_\perp \neq 0$, the time can be expressed as a function of the scale factor,

$$t(a) = \pm t_{\text{BC}} \int_{a/a_{\text{max}}}^1 \frac{x^{11} dx}{\sqrt{1-x^{18}}} \quad \text{where} \quad t_{\text{BC}} = \frac{\sqrt{3}}{|c_M|} a_{\text{max}}^{12}, \quad \forall a \leq a_{\text{max}} = \left| \frac{c_M}{c_m} \right|^{1/9}. \quad (2.47)$$

Formally, this solution describes a Big Bang at $t = -t_{\text{BC}}$ that initiates an expansion era. The latter stops when the scale factor reaches its maximum a_{max} at $t = 0$. The universe then contracts till $t = t_{\text{BC}}$, where a Big Crunch occurs. As before, the last era of this evolution can be trusted for $t \lesssim t_{\text{BC}}$, where the solution (2.47) is asymptotic to (2.46). Thus, we have shown that for IBC such that $z_0 \gg 1$ and $c_\phi = 0$, the dynamics is attracted to the MDS described by the Big Crunch solution of Eq. (2.46) with $c_\phi = c_\perp = 0$ and $3H^2 = c_M/a^{24}$.

(ii) $c_\phi \neq 0$:

We consider IBC such that $z_0 \gg 1$, with generic $c_\phi \neq 0$. The sign of $n_V \neq 0$ is arbitrary and we want again to find the basins of attraction of the dynamics. Instead of Eq. (2.40), it is more convenient to use the combination of Eq. (2.40) and Eq. (2.41),

$$\dot{H} = -\frac{1}{2}\dot{\phi}^2 - \frac{1}{2}\dot{\phi}_\perp^2, \quad (2.48)$$

that becomes, utilizing the expressions (2.43) for $\dot{\phi}$ and $\dot{\phi}_\perp$:

$$\dot{H} = -\frac{1}{3} \left(6H - \frac{c_\phi}{a^3} \right)^2 - \frac{c_\perp^2}{a^6}. \quad (2.49)$$

Introducing the quantity σ ,

$$\sigma = \frac{9}{4c_\phi} a^2 \dot{a} = \frac{3}{4c_\phi} (a^3)' = \frac{9}{4c_\phi} a^3 H, \quad (2.50)$$

Eq. (2.49) takes the form:

$$\frac{\sigma d\sigma}{\sigma^2 - \sigma + \frac{3}{16}(1 + 3(c_\perp/c_\phi)^2)} = -9 \frac{da}{a}, \quad (2.51)$$

valid in the $z \gg 1$ limit. In the same limit the field equations become:

$$\begin{cases} \frac{1}{\frac{1}{3}(\dot{M}/M)^2 + \frac{1}{2}\dot{\phi}_\perp^2 - 3} (\ddot{M}/M)^\circ - \dot{M}/M - 6 = 0 \\ \frac{1}{\frac{1}{3}(\dot{M}/M)^2 + \frac{1}{2}\dot{\phi}_\perp^2 - 3} \ddot{\phi}_\perp^\circ - \dot{\phi}_\perp^\circ = 0 \end{cases} \quad (2.52)$$

where the signs of n_V and the fractions in front of the second derivatives must be the same, so that the right hand side of Eq. (2.25) is positive. This condition is required to study the

evolutions in real time (and not Euclidean). This differential system is easily shown to be equivalent to

$$\begin{cases} 3(\dot{M}/M)^\circ = (\dot{M}/M + 6)\mathcal{P}(\dot{M}/M) \\ \dot{\phi}_\perp = \sqrt{2} \frac{c_\perp}{c_\phi} (\dot{M}/M + 6) \end{cases} \quad (2.53)$$

where c_\perp/c_ϕ is an arbitrary constant denoted this way to make contact with the conventions of Eq. (2.43), and the polynomial

$$\mathcal{P}(x) = [1 + 3(c_\perp/c_\phi)^2]x^2 + 36(c_\perp/c_\phi)^2x + 9[12(c_\perp/c_\phi)^2 - 1] \text{ is of the sign of } n_V. \quad (2.54)$$

Using the first equation in (2.43), one obtains

$$\sigma = \frac{9}{4} \frac{1}{\dot{M}/M + 6}, \quad (2.55)$$

that can be used to identify consistently (2.51) with the first equation of (2.52). By the way, the real time condition translates to the condition that n_V and the denominator of the left hand side of (2.51) have a common sign. We are going to solve this equation separately for $n_V > 0$ and $n_V < 0$.

- *For $n_V > 0$:*

i) When $(c_\perp/c_\phi)^2 > 1/9$, the denominator of (2.51) has no real root. Integrating, one obtains

$$\frac{a}{a_0} = \frac{\exp\left(-\frac{2}{9\sqrt{9(c_\perp/c_\phi)^2-1}} \tan^{-1}\left(\frac{4\sigma-2}{\sqrt{9(c_\perp/c_\phi)^2-1}}\right)\right)}{(\sigma^2 - \sigma + \frac{3}{16}(1 + 3(c_\perp/c_\phi)^2))^{1/18}}, \quad (2.56)$$

that is used to draw σ versus a on Fig. 3a. Suppose $c_\phi > 0$. If at some time $\sigma > 0$, the definition of σ in Eq. (2.50) implies that a increases. Thus, the scale factor reaches a maximum a_{\max} where σ vanishes. We then pass into the $\sigma < 0$ lower half plane where a decreases and converges to 0. Since $\sigma \rightarrow -\infty$ and $a \rightarrow 0$, one has $\dot{a} \rightarrow -\infty$, which is interpreted as a Big Crunch occurring at a finite time t_{BC} . Similar arguments apply when $c_\phi < 0$: If $\sigma < 0$, the scale factor increases up to a_{\max} before converging to 0. This corresponds to a Big Crunch occurring at some finite time t_{BC} . To summarize, when the IBC are such that $c_\phi > 0$ ($c_\phi < 0$), the system follows the curve on Fig. 3a from up to down (down to up). The universe starts from a Big Bang, reaches a maximum size, and ends with a Big Crunch. Of course, only the parts of this evolution consistent with the hypothesis

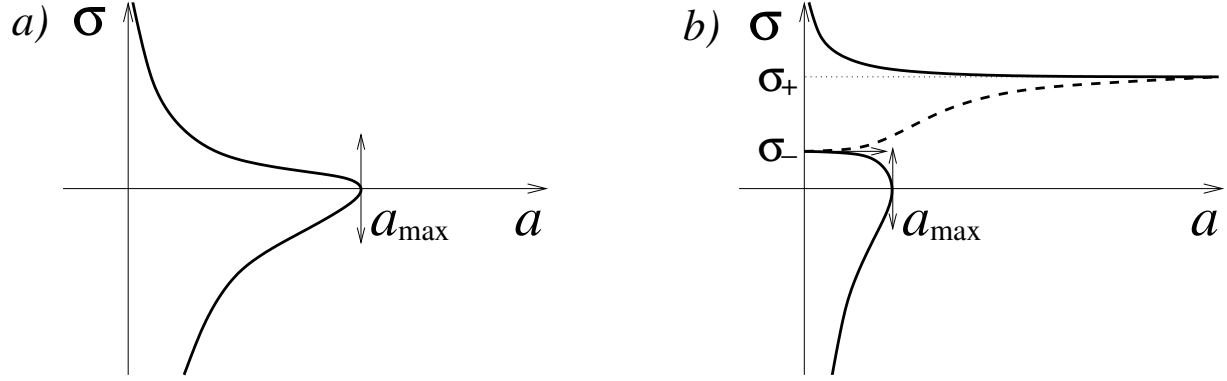


Figure 3: σ , defined in Eq. (2.50) versus the scale factor a . Fig. a) corresponds to $(c_\perp/c_\phi)^2 > 1/9$ and Fig. b) corresponds to $(c_\perp/c_\phi)^2 \leq 1/9$. The solid lines are associated to the case $n_V > 0$, while the dashed one is associated to the case $n_V < 0$.

$z \gg 1$ can be trusted. In particular, when we approach t_{BC} , one has $\sigma \rightarrow -s\infty$, where $s = \text{sign}(c_\phi)$, and Eq. (2.56) implies,

$$a(t) \simeq (t_{\text{BC}} - t)^{1/12} \times ((16/3)|c_\phi| a_0^9 e^{s9\pi/2})^{1/12}, \quad t \lesssim t_{\text{BC}}. \quad (2.57)$$

This result can be used to show that the integral in M in Eqs (2.43) is bounded when $t \rightarrow t_{\text{BC}}$ so that a consistent run away $e^z \propto 1/a^5 \rightarrow +\infty$ is found. An important remark then follows. The behavior (2.57) implies that c_ϕ/a^3 is dominated by H in Eq. (2.43), showing that the dynamics for $(c_\perp/c_\phi)^2 > 1/9$ is always attracted by the solution for $c_\phi = c_\perp = 0$.

ii) When $(c_\perp/c_\phi)^2 \leq 1/9$, the left hand side of Eq. (2.51) has two real roots. Integrating, one has

$$\text{for } \left(\frac{c_\perp}{c_\phi}\right)^2 < \frac{1}{9} : \quad \frac{a}{a_0} = \frac{|\sigma - \sigma_-|^{\frac{\sigma_-}{9(\sigma_+ - \sigma_-)}}}{|\sigma - \sigma_+|^{\frac{\sigma_+}{9(\sigma_+ - \sigma_-)}}}, \quad \sigma_\pm = \frac{1}{2} \pm \frac{1}{4}\sqrt{1 - 9(c_\perp/c_\phi)^2}, \quad (2.58)$$

$$\text{for } \left(\frac{c_\perp}{c_\phi}\right)^2 = \frac{1}{9} : \quad \frac{a}{a_0} = \frac{e^{\frac{1}{18(\sigma - 1/2)}}}{|\sigma - \frac{1}{2}|^{1/9}}, \quad \sigma_\pm = \frac{1}{2}, \quad (2.59)$$

where $\sigma_- < \sigma < \sigma_+$. Fig. 3b shows σ as a function of a .⁵ When $c_\phi > 0$, the arguments used in case i) apply on the branch $\sigma < \sigma_-$, where the system evolves from up to down. The scale factor grows up to a_{max} before converging to 0. Similarly, when $c_\phi < 0$ and the system follows the branch $\sigma > \sigma_+$, the scale factor converges to 0. In both cases, a Big Crunch

⁵To be precise, the slope at the point $(a, \sigma) = (0, \sigma_-)$ is infinite when $(1 - 4/361)/9 < (c_\perp/c_\phi)^2 \leq 1/9$, finite when $(c_\perp/c_\phi)^2 = (1 - 4/361)/9$ and vanishing when $(c_\perp/c_\phi)^2 < (1 - 4/361)/9$.

occurs at t_{BC} and the behavior (2.57), with $s = 0$, is valid. The dynamics is again attracted by the solution associated to $c_\phi = c_\perp = 0$.

Some new features occur for $c_\phi < 0$, when $\sigma < \sigma_-$, (see Fig. 3b). The scale factor increases up to its maximum a_{max} before converging to 0, while $\sigma \rightarrow \sigma_-$, implying $\dot{a} \rightarrow -\infty$. A Big Crunch at a finite time t_{BC} occurs but since $\sigma \rightarrow \sigma_-$, one has

$$a(t) \simeq (t_{\text{BC}} - t)^{1/3} \times ((4/3)|c_\phi|\sigma_-)^{1/3}, \quad t \lesssim t_{\text{BC}}. \quad (2.60)$$

However, this behavior can be trusted as long as $e^z \propto aM \propto |t_{\text{BC}} - t|^{\frac{3}{4\sigma_-} - \frac{5}{3}} \rightarrow +\infty$. This condition implies that the scaling $(t_{\text{BC}} - t)^{1/3}$ exists when

$$c_\phi < 0 \quad \text{and} \quad \frac{1}{9}(1 - 1/25) < \left(\frac{c_\perp}{c_\phi}\right)^2 \leq \frac{1}{9}. \quad (2.61)$$

In this case, the evolution is not attracted by the solution $c_\phi = c_\perp = 0$. Actually, Eq. (2.43) can be rewritten as

$$\alpha\phi \simeq -\left(2 - \frac{3}{4\sigma_-}\right) \ln |t_{\text{BC}} - t| + c \rightarrow +\infty \quad \text{when} \quad t \rightarrow t_{\text{BC}}, \quad (2.62)$$

so that the integration constant c is negligible. There is thus a new basin of attraction, to the solution (2.60) with $c = 0$ *i.e.* such that

$$3H^2 \simeq \frac{c_m^2}{a^6} \quad \text{where} \quad c_m = \frac{1}{27} c_\phi^2 \sigma_-^2, \quad c = 0. \quad (2.63)$$

As for the previous Big Crunch behavior, the divergence of M (and T) is only formal since the present analysis supposes $M < M_H$, where a Hagedorn-like transition occurs. On the contrary, for $(c_\perp/c_\phi)^2$ smaller than the lower bound of the range (2.61), we formally have $e^z \propto |t_{\text{BC}} - t|^{\frac{3}{4\sigma_-} - \frac{5}{3}} \rightarrow 0$, which shows that the system actually exits from the regime $z \gg 1$ we started with. Similarly, if $c_\phi > 0$ and the system evolves along the branch $\sigma > \sigma_+$ of Fig. 3b, one has $a \rightarrow +\infty$ and $\sigma \rightarrow \sigma_+$. This implies $a \simeq t^{1/3} \times ((4/3)c_\phi\sigma_+)^{1/3}$ and $t \rightarrow +\infty$. However, for large positive z , this implies $e^z \propto aM \propto t^{\frac{3}{4\sigma_+} - \frac{5}{3}} \rightarrow 0$, which shows that the system actually exits in the regime $z \gg 1$.

Summary for $n_V > 0$:

We have shown analytically that when $z_0 \gg 1$, either $z \rightarrow +\infty$ or z quits the regime $z \gg 1$. To study the dynamics at finite z , we use again a code that implements Eqs (2.26). The phases of expansion are simulated when a positive increment for λ is chosen. For generic

IBC, we observe that the right hand side of the Friedmann equation (2.25) changes sign at some finite $a = a_{\max}$. This actually means that the scale factor has reached a maximum, (the Hubble parameter H vanishes), and that the evolution enters into a phase of contraction. Such eras are simulated by choosing a negative increment for λ and we observe that for generic IBC, z ends by running away. As a conclusion, the generic IBC admit two basins of attraction to MDS. The evolutions converge either to the solution where $c_\phi = c_\perp = 0$ *i.e.* such that $3H^2 = c_M^2/a^{24}$, or to the solution with $c = 0$ and $3H^2 = c_m^2/a^6$.

- *For $n_V < 0$:*

For IBC such that $z_0 \gg 1$, the real time condition translates to $\sigma_- < \sigma < \sigma_+$, (see Fig. 3b). If $c_\phi > 0$, the definition of σ in Eq. (2.51) reaches $\dot{a} > 0$. It follows that $a \rightarrow +\infty$ and $\sigma \rightarrow \sigma_+$. Thus, $a \simeq t^{1/3} \times ((4/3)c_\phi\sigma_+)^{1/3}$ where $t \rightarrow +\infty$, and $e^z \propto aM \propto t^{\frac{3}{4\sigma_+} - \frac{5}{3}} \rightarrow 0$. We conclude that the system exits the regime $z \gg 1$. When $-1/15 < n_V/n_T < 0$ (*Case (b)*), this result is compatible with Sect. 2.3, where we found that the expanding evolutions satisfy $z \rightarrow z_c$.

If on the contrary $c_\phi < 0$, one has $\dot{a} < 0$. It follows from Fig. 3b that $a \rightarrow 0$ and $\sigma \rightarrow \sigma_-$, so that $\dot{a} \rightarrow -\infty$. The dynamics is attracted by the Big Crunch solution (2.60) *i.e.* (2.63) that can be trusted when the conditions (2.61) are satisfied. For smaller values of $(c_\perp/c_\phi)^2$, the system exits the regime $z \gg 1$. These conclusions are compatible with the numerical simulations of the contracting evolutions described at the end of Sect. 2.4, for generic IBC in *Case (b)*. We found there that either $z \rightarrow +\infty$, as expected from the existence of the Big Crunch solution (2.60), or z oscillates with a larger and larger amplitude, implying that when z is large and positive, it can exit the regime $z \gg 1$.

• $n_V = 0$, $z \gg 1$:

We argued in Section 2.3 that when $-1/15 < n_V/n_T < 0$, the expanding evolutions are such that z is attracted by a critical value z_c . Since $z_c \rightarrow +\infty$ when $n_V/n_T \rightarrow 0_-$, we expect the expanding solutions for $n_V = 0$ to satisfy $z(t) \rightarrow +\infty$. Actually, this run away may be natural since the potential in Eq. (2.27) decreases linearly for large positive z , $V \sim -zn_T c_4^0$. In fact, a numerical study of the differential system (2.26) with $n_V = 0$ confirms that for generic IBC, z enters the regime $z \gg 1$ when we simulate the expanding solutions. On the other hand, for the evolutions where the scale factor decreases, we find either $z \rightarrow +\infty$ or z oscillates with a larger and larger amplitude. In all these cases, we are thus invited to

consider analytically the system in the regime $z \gg 1$.

In this limit, the thermal effective potential for ϕ in Eq. (2.16) vanishes (up to exponentially small terms in z). Both ϕ and ϕ_\perp are flat directions, and their kinetic energies involve two constants c_ϕ, c_\perp , determined by the IBC,

$$\alpha \dot{\phi} = \frac{c_\phi}{a^3}, \quad \dot{\phi}_\perp = \sqrt{2} \frac{c_\perp}{a^3}. \quad (2.64)$$

The energy density in the Friedmann Eq. (2.12) is $\rho \simeq 3T^4 n_T c_4^o$ and, using (2.39), one has

$$3H^2 = \frac{c_r^2}{a^4} + \frac{c_m^2}{a^6} \quad \text{where} \quad c_r^2 = 3n_T c_4^o (a_0 T_0)^4, \quad c_m^2 = c_\perp^2 + \frac{c_\phi^2}{3}. \quad (2.65)$$

This equation admits two monotonic solutions (see Eq. (2.32)), mapped to one another under time reversal. However, one can only trust them as long as $z \gg 1$.

- For the expanding evolutions, since $a(t) \rightarrow +\infty$ when $t \rightarrow +\infty$, Eq. (2.64) implies that $M(t)$ converges to a constant M_0 . It follows that $e^z = M/T \propto a \rightarrow +\infty$, showing the validity of the solution for large enough time. We conclude that for generic IBC, the expanding evolutions are attracted to the RDS, Eq. (2.33), with $M = M_0$.

- For the contracting evolutions, depending on c_ϕ and c_\perp , we have

$$a(t) \simeq (t_{\text{BC}} - t)^{1/3} \times (3c_m^2)^{1/6}, \quad M \simeq M_0 \left(1 - \frac{t}{t_{\text{BC}}}\right)^{-\frac{1}{[1+3(c_\perp/c_\phi)^2]^{1/2}}}, \quad t \lesssim t_{\text{BC}}, \quad (2.66)$$

if the constraints

$$c_\phi > 0 \quad \text{and} \quad \left(\frac{c_\perp}{c_\phi}\right)^2 < \frac{8}{3} \quad (2.67)$$

are satisfied, (so that $z \rightarrow +\infty$ when $t \rightarrow t_{\text{BC}}$). Alternatively, if $c_\phi = c_\perp = 0$, the solution

$$a(t) \simeq \sqrt{t_{\text{BC}} - t} \times \left(\frac{4}{3} c_r^2\right)^{1/4}, \quad M \simeq M_0 \quad (2.68)$$

is also consistent with a run away of z . For all other values of c_ϕ and c_\perp , z exits the regime $z \gg 1$. Our conclusions for the contracting evolutions are that there are two sets of generic IBC. The first gives rise to a run away of z that corresponds to the Big Crunch solutions (2.66) or (2.68). The second yields an oscillation regime in z , with larger and larger amplitude.

3 Susy breaking involving $n = 2$ internal directions

In this section we extend our analysis to models where the supersymmetry breaking is induced by $n = 2$ Scherk-Schwarz circles $S^1(R_4) \times S^1(R_5)$ in the 4th and 5th internal dimensions as in Ref. [14]. Thus, we are considering the heterotic or type II superstrings on

$$S^1(R_0) \times T^3 \times S^1(R_4) \times S^1(R_5) \times \mathcal{M}_4, \quad (3.1)$$

where \mathcal{M}_4 is either T^4 or $K3 \sim T^4/\mathbb{Z}_2$. As shown in [14], such models allow cosmological evolutions that describe radiation dominated eras, with stabilized complex structure modulus R_5/R_4 [14]. The present analysis could certainly be extended to the other classes of models described in [14], where asymmetric Scherk-Schwarz compactifications in the directions 4 and/or 5 are considered. These models would involve run away solutions in the spirit of the *Cases (c)* and *(a)* for $n = 1$.

R_0, R_4, R_5 are restricted to be large enough in order to avoid Hagedorn-like phase transitions. In this regime, the free energy and pressure are computed in Ref. [14] :

$$P = T^4 p(z, Z) \quad \text{where} \quad e^z = \frac{M}{T}, \quad T = \frac{1}{2\pi R_0 \sqrt{\text{Re}S}}, \quad M = \frac{1}{2\pi \sqrt{R_4 R_5} \sqrt{\text{Re}S}}, \quad e^Z = \frac{R_5}{R_4}. \quad (3.2)$$

T is the temperature in the Einstein frame, M is the supersymmetry breaking scale which is given in terms of the geometric mean of the moduli R_4 and R_5 , and Z parametrizes the complex structure modulus R_5/R_4 . The quantity $p(z, Z)$ is a linear sum of functions, with model-dependent integer coefficients,

$$p(z, Z) = n_T p_{100}(z, Z) + n_{010} p_{010}(z, Z) + n_{001} p_{001}(z, Z) + n_{111} p_{111}(z, Z), \quad (3.3)$$

where n_T is again the number of massless states and

$$p_{\tilde{g}_0 \tilde{g}_4 \tilde{g}_5} = \frac{2}{\pi^3} \sum_{\tilde{m}_0, \tilde{m}_4, \tilde{m}_5} \frac{e^{4z}}{[(2\tilde{m}_0 + \tilde{g}_0)^2 e^{2z} + (2\tilde{m}_4 + \tilde{g}_4)^2 e^{-Z} + (2\tilde{m}_5 + \tilde{g}_5)^2 e^Z]^3}. \quad (3.4)$$

3.1 Gravitational and moduli equations

The low energy effective action (2.7) with the kinetic term of R_5 taken into account can be written in terms of redefined fields as :

$$S = \int d^4x \sqrt{-g} \left[\frac{1}{2} R - \frac{1}{2} \left((\partial\phi)^2 + (\partial\phi_\perp)^2 + \frac{1}{2} (\partial Z)^2 \right) + P \right], \quad (3.5)$$

where

$$\phi := \phi_D - \ln \sqrt{R_4 R_5}, \quad \phi_\perp := \phi_D + \ln \sqrt{R_4 R_5}, \quad P = M^4 e^{-4z} p(z, Z), \quad M = \frac{e^\phi}{2\pi}. \quad (3.6)$$

For evolutions satisfying the metric ansatz (2.11), the Friedmann equation is

$$3H^2 = \frac{1}{2}\dot{\phi}^2 + \frac{1}{4}\dot{Z}^2 + \frac{1}{2}\dot{\phi}_\perp^2 + T^4 r \quad \text{where} \quad r(z, Z) = 3p - p_z, \quad (3.7)$$

while the equation for the scale factor,

$$2\dot{H} + 3H^2 = -\frac{1}{2}\dot{\phi}^2 - \frac{1}{4}\dot{Z}^2 - \frac{1}{2}\dot{\phi}_\perp^2 - T^4 p, \quad (3.8)$$

can be replaced by the conservation of the energy-momentum in a form similar to Eq. (2.19),

$$\dot{\rho} + 3H(\rho + P) + \dot{\phi} T^4 (3p - r) + \dot{Z} T^4 p_z = 0. \quad (3.9)$$

For the scalar fields, one has

$$\ddot{\phi} + 3H\dot{\phi} = T^4 (3p - r), \quad (3.10)$$

$$\ddot{Z} + 3H\dot{Z} = T^4 2p_z, \quad (3.11)$$

$$\ddot{\phi}_\perp + 3H\dot{\phi}_\perp = 0 \quad \implies \quad \dot{\phi}_\perp = \sqrt{2} \frac{c_\perp}{a^3}. \quad (3.12)$$

We proceed by introducing the variable λ of (2.20) as in Section 2.2 to express

$$\overset{\circ}{\phi} = \mathcal{A}(z, Z)\overset{\circ}{z} + \mathcal{B}(z, Z)\overset{\circ}{Z} - 1 \quad \text{where} \quad \mathcal{A}(z, Z) = \frac{4r - r_z}{3(r + p)}, \quad \mathcal{B}(z, Z) = -\frac{r_z + p_z}{3(r + p)}, \quad (3.13)$$

that reaches, by integration,

$$M = \frac{e^{\mathcal{F}(\ln a)}}{a} \quad \text{where} \quad \overset{\circ}{\mathcal{F}}(\lambda) := \mathcal{A}(z, Z)\overset{\circ}{z} + \mathcal{B}(z, Z)\overset{\circ}{Z}. \quad (3.14)$$

This result can be used to rewrite the Friedmann Eq. (3.7) in either of the following forms,

$$3H^2 = \frac{3}{5} H^2 \left((\mathcal{A}\overset{\circ}{z} + \mathcal{B}\overset{\circ}{Z} - 1)^2 + \frac{1}{2} \overset{\circ}{Z} - 1 \right)^2 + \frac{6}{5} \frac{e^{4[\mathcal{F}(\ln a) - z]}}{a^4} r(z, Z) + \frac{6}{5} \frac{c_\perp^2}{a^6}, \quad (3.15)$$

or

$$H^2 = T^4 \frac{r(z, Z)}{3 - \mathcal{K}(z, \overset{\circ}{z}, Z, \overset{\circ}{Z}, \overset{\circ}{\phi}_\perp)} \quad \text{where} \quad \mathcal{K} = \frac{1}{2} \left(\mathcal{A}(z, Z)\overset{\circ}{z} + \mathcal{B}(z, Z)\overset{\circ}{Z} - 1 \right)^2 + \frac{1}{4} \overset{\circ}{Z}^2 + \frac{1}{2} \overset{\circ}{\phi}_\perp^2, \quad (3.16)$$

while the equations for the scalars become

$$\begin{cases} \frac{r}{3-\mathcal{K}} \left(\mathcal{A}^{\circ\circ} \ddot{z} + \mathcal{B}^{\circ\circ} \ddot{Z} + \mathcal{A}_z \ddot{z}^2 + (\mathcal{A}_Z + \mathcal{B}_z) \ddot{z} \dot{Z} + \mathcal{B}_Z \ddot{Z}^2 \right) + \frac{r-p}{2} (\mathcal{A}^{\circ} \dot{z} + \mathcal{B}^{\circ} \dot{Z}) + V_z^{(z)} = 0 \\ \frac{r}{3-\mathcal{K}} \ddot{Z} + \frac{r-p}{2} \dot{Z} + V_Z^{(Z)} = 0 \\ \frac{r}{3-\mathcal{K}} \ddot{\phi}_{\perp} + \frac{r-p}{2} \dot{\phi}_{\perp} = 0 \end{cases} \quad (3.17)$$

where we have defined

$$V_z^{(z)}(z, Z) = \frac{r-5p}{2} \quad \text{and} \quad V_Z^{(Z)}(z, Z) = -2p_Z. \quad (3.18)$$

3.2 Attraction to the RDS

A wide range of behaviors can emerge from this system of differential equations. However, in the spirit of Section 2.3, we choose to look for cosmological solutions with stabilized complex structures $e^z = R_0/\sqrt{R_4 R_5}$ and $e^Z = R_5/R_4$ *i.e.* satisfying $(z, Z) \equiv (z_c, Z_c)$. From the system (3.17), such solutions exist if the following conditions are simultaneously satisfied,

$$V_z^{(z)}(z_c, Z_c) = 0 \quad \text{i.e.} \quad 2p(z_c, Z_c) + p_z(z_c, Z_c) = 0, \quad (3.19)$$

$$V_Z^{(Z)}(z_c, Z_c) = 0 \quad \text{i.e.} \quad p_Z(z_c, Z_c) = 0. \quad (3.20)$$

These equations define two sets of curves in the (z, Z) -plane, whose intersections determine the critical points (z_c, Z_c) . Supposing such a solution exists, the corresponding cosmological evolution is characterized by $\dot{\mathcal{F}} = 0$ in Eq. (3.14) and thus

$$M(t) = e^{z_c} T(t) = \frac{e^{\mathcal{F}}}{a(t)} \quad \text{where} \quad \mathcal{F} = \text{cst.}, \quad R_5(t) = e^{Z_c} R_4(t), \quad (3.21)$$

together with the Friedmann Eq. (3.15),

$$3H^2 = \frac{c_r^2}{a^4} + \frac{c_m^2}{a^6} \quad \text{where} \quad c_r^2 = 6 e^{4(\mathcal{F}-z_c)} p(z_c, Z_c), \quad c_m^2 = \frac{6}{5} c_{\perp}^2. \quad (3.22)$$

The latter is qualitatively identical to the one found for $n = 1$ in *Case (b)*. The evolution with expanding scale factor is asymptotic to the critical solution $(z, Z) \equiv (z_c, Z_c)$, $c_m = 0$, where $a(t)$ is given in (2.33).

To study the local stability around a critical solution with $c_m = 0$, we define small perturbations around it,

$$z(\lambda) = z_c + \varepsilon(\lambda), \quad Z(\lambda) = Z_c + E(\lambda), \quad \overset{\circ}{\phi}_\perp(\lambda) = \overset{\circ}{\varepsilon}_\perp(\lambda), \quad (3.23)$$

and consider the system (3.17) at first order in ε , E and $\overset{\circ}{\varepsilon}_\perp$. The latter can be brought into the form

$$\begin{cases} \overset{\circ\circ}{\varepsilon} + \overset{\circ}{\varepsilon} + \xi \varepsilon + \omega E = 0 & \text{where } \xi = \frac{9p(4p-p_{zz})+2p_{zz}^2}{2p(26p+p_{zz})} \Big|_{(z_c, Z_c)}, \quad \omega = \frac{p_{zz}(2p_{zz}-9p)}{2p(26p+p_{zz})} \Big|_{(z_c, Z_c)}, \\ \overset{\circ\circ}{E} + \overset{\circ}{E} + \Xi E + \Omega \varepsilon = 0 & \text{where } \Xi = -\frac{p_{zz}}{p} \Big|_{(z_c, Z_c)}, \quad \Omega = -\frac{p_{zz}}{p} \Big|_{(z_c, Z_c)}, \\ \overset{\circ\circ}{\varepsilon}_\perp + \overset{\circ}{\varepsilon}_\perp = 0. \end{cases} \quad (3.24)$$

To proceed, we specialize on the set of models whose R-symmetry charges coupled to the winding and momentum numbers in the directions 4 and 5 are identical. For this set, it is shown in [14] that the function p in Eq. (3.3) takes the form,

$$p(z, Z) = n_T[p_{100} + p_{111}] + n_V[p_{010} + p_{001}], \quad -1 \leq \frac{n_V}{n_T} \leq 1, \quad (3.25)$$

and that the range of n_V/n_T can be divided in four phases whose frontiers are given by $\{-0.215, -1/31, 0\}$ [14].

- For $n_V/n_T \lesssim -0.215$, the constraint (3.19) has no solution.
- For $-0.215 \lesssim n_V/n_T < -1/31$, the constraint (3.19) defines a non trivial curve, while the condition (3.20) is satisfied on the axis $Z = 0$. The two loci intersect at a point $(z_c, 0)$.
- For $-1/31 < n_V/n_T < 0$, the constraint (3.19) defines a non trivial curve, while the condition (3.20) corresponds to 3 curves, one of which being the axis $Z = 0$. There is a unique intersection point $(z_c, 0)$.
- For $0 < n_V/n_T$, the constraint (3.19) has no solution.

Thus, a solution to the equations of motion with constant complex structures (z_c, Z_c) exists when $-0.215 \lesssim n_V/n_T < 0$.

The class of models we consider have a symmetry $R_4 \leftrightarrow R_5$ that translates to $Z \leftrightarrow -Z$. This implies $p_{zZ}(z_c, 0) = 0$ and

$$\xi = \frac{9}{2} \frac{4p - p_{zz}}{26p + p_{zz}} \Big|_{(z_c, Z_c)}, \quad \omega = 0, \quad \Xi = -\frac{p_{ZZ}}{p} \Big|_{(z_c, Z_c)}, \quad \Omega = 0. \quad (3.26)$$

As in the $n = 1$ case, one finds numerically that ξ and Ξ are always positive. They increase from 0 to $+\infty$ when n_T/n_V varies from $\simeq -0.215$ to 0, which shows that $\varepsilon(\lambda)$, $E(\lambda)$ (and $\overset{\circ}{\varepsilon}_\perp(\lambda)$) are always converging to zero. This means that in the class of models under investigation, when a solution (z_c, Z_c) exists, it is stable under small perturbations.

A numerical study of the exact system (3.17) shows that for generic IBC $(z_0, \overset{\circ}{z}_0, Z_0, \overset{\circ}{Z}_0, \overset{\circ}{\phi}_{\perp 0})$ such that the right hand side of Eq. (3.16) is positive, the expanding cosmological solutions satisfy $(z(\lambda), Z(\lambda)) \rightarrow (z_c, Z_c)$ when $\lambda \rightarrow +\infty$. We conclude that the dynamics of the expanding evolutions is attracted by the RDS described by the critical solution $(z, Z) \equiv (z_c, Z_c)$, $\dot{\phi}_\perp \equiv 0$ *i.e.* $3H^2 = c_r^2/a^4$ (Eq. (3.21)).

4 An inflation era ?

We have described how cosmological evolutions can be determined by the R-charges of the massless spectrum. In particular, we found that the dynamics can be attracted to a radiation era, with stabilized complex structures. In standard cosmology, such an era follows a period of inflation introduced to explain the observed flatness and homogeneity of our universe, or solve the topological defect problem. It is thus important to analyze if our attraction mechanism admits periods of accelerated cosmology.

Using the Einstein equations, the inflation condition $\ddot{a} > 0$ or $\dot{H} + H^2 > 0$ can be translated into an inequality between the thermal sources ρ , P and the scalar kinetic energies (take $\dot{Z} = 0$ for $n = 1$),

$$\frac{1}{2} \left(P + \frac{1}{3} \rho \right) < -\frac{1}{3} \left(\dot{\phi}^2 + \frac{1}{2} \dot{Z}^2 + \dot{\phi}_\perp^2 \right). \quad (4.1)$$

Using the variable λ and Eq. (2.25) (or (3.16)), we obtain

$$p - \frac{1}{6} p_z < -2 \left(p - \frac{1}{3} p_z \right) \frac{1}{\frac{3}{\kappa} - 1} \leq 0, \quad (4.2)$$

where there are two conditions in one: First, $p - p_z/6$ has to be negative, and second it has to be negative enough compared to the right hand side that depends on the kinetic energies.

For the classes of models we considered for $n = 1$ (see Eq. (2.4)) and $n = 2$ (see Eq. (3.25)), one can show that a necessary condition to have $p - p_z/6 < 0$ is $n_V < 0$. We thus carry on our discussion in this case. It then follows that the quantity $p - p_z/3$ in (4.2) is always positive. Altogether, a necessary and sufficient condition for an accelerated cosmology is:

$$n_V < 0 \quad \text{and} \quad \mathcal{K} < \mathcal{I} \quad \text{where} \quad \mathcal{I} = -\frac{3p - \frac{1}{2}p_z}{p - \frac{1}{2}p_z}. \quad (4.3)$$

Actually, when deriving (4.3) from (4.2), one also finds that $\mathcal{I} \leq 3$. However, this is an empty constraint since the function \mathcal{I} satisfies $\mathcal{I} \leq 1$.

For a susy breaking with $n = 1$

From (4.3), in order to have solutions, we need $\mathcal{I}(n_V/n_T, z) > 0$. This condition defines a domain in the $(n_V/n_T, z)$ -plane where, by definition, $\mathcal{I} = 0$ on the boundary. However, it happens that $\mathcal{I} \lesssim 1$ in the interior, as can be seen on Fig. 4a. We can thus estimate the

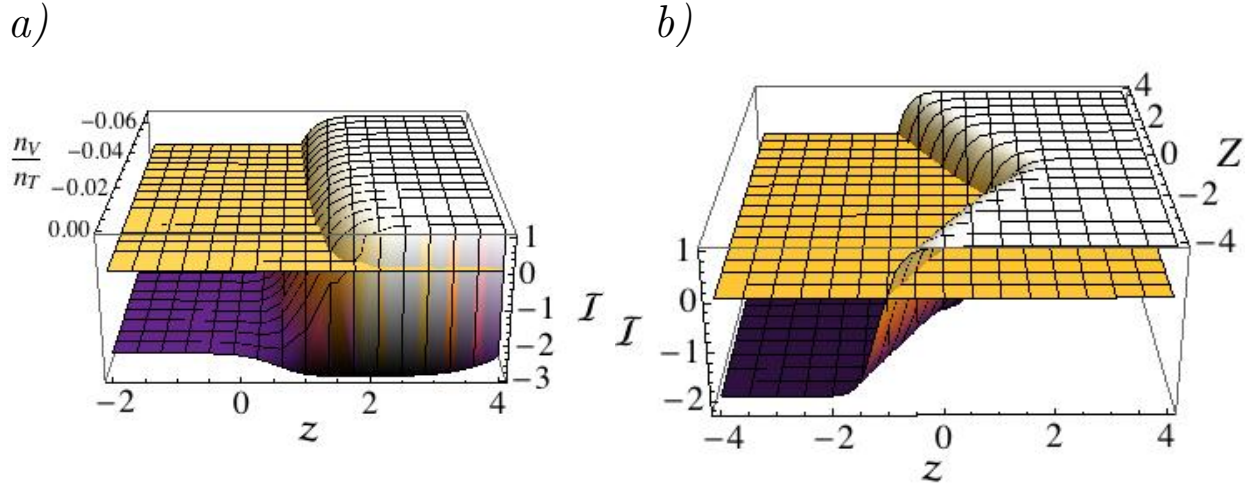


Figure 4: *For one modulus z (Fig. a): In the domain $\mathcal{I} > 0$ of the $(n_V/n_T, z)$ -plane ($-1/15 < n_V/n_T < 0$), \mathcal{I} can be approximated by 1, its value for $z \gg 1$. For two moduli (z, Z) (Fig. b): For any fixed $-0.215 \lesssim n_V/n_T < 0$, in the domain $\mathcal{I} > 0$ of the (z, Z) -plane, \mathcal{I} can be approximated by 1, its asymptotic value in the sectors II, III, IV, V defined in Eq. (4.9). Fig. b), is drawn for $n_V/n_T = -1/63$.*

e -fold number by considering z large and positive. In particular, when $-1/15 < n_V/n_T < 0$, the z 's allowing periods of accelerated cosmology are always larger than z_c . When $z \gg 1$, the condition (4.3) simplifies to

$$\mathcal{K} \simeq \frac{1}{3} (\dot{M}/M)^2 + (c_\perp/c_\phi)^2 (\dot{M}/M + 6)^2 \lesssim 1, \quad (4.4)$$

where we have used the second equation of (2.53). An era of acceleration exists if $(c_\perp/c_\phi)^2 <$

1/33. It begins and ends at most when \dot{M}/M saturates the inequality (4.4) *i.e.* equals

$$l_{b,e} = \frac{-18(c_{\perp}/c_{\phi})^2 \pm \sqrt{3[1 - 33(c_{\perp}/c_{\phi})^2]}}{1 + 3(c_{\perp}/c_{\phi})^2}. \quad (4.5)$$

To find the scale factors a_b and a_e when this arises, we integrate the first equation of (2.53),

$$\ln \left(\frac{a}{a_0} \right) \equiv \lambda - \lambda_0 = \gamma \ln |\dot{M}/M + 6| + \gamma_+ \ln |\dot{M}/M - l_+| + \gamma_- \ln |\dot{M}/M - l_-|, \quad (4.6)$$

where

$$l_{\pm} = \frac{-18(c_{\perp}/c_{\phi})^2 \pm 3\sqrt{1 - 9(c_{\perp}/c_{\phi})^2}}{1 + 3(c_{\perp}/c_{\phi})^2}, \quad (4.7)$$

$$\gamma = \frac{3}{1 + 3(c_{\perp}/c_{\phi})^2} \frac{1}{(l_+ + 6)(l_- + 6)}, \quad \gamma_{\pm} = \pm \frac{3}{1 + 3(c_{\perp}/c_{\phi})^2} \frac{1}{(l_{\pm} + 6)(l_+ - l_-)}.$$

Inserting the particular values (4.5) in (4.6), the maximum e -fold number is found to be

$$e(c_{\perp}/c_{\phi}) = \ln \frac{a_e}{a_b} = \gamma \ln \left| \frac{l_e + 6}{l_b + 6} \right| + \gamma_+ \ln \left| \frac{l_e - l_+}{l_b - l_+} \right| + \gamma_- \ln \left| \frac{l_e - l_-}{l_b - l_-} \right|, \quad (4.8)$$

that satisfies $e(c_{\perp}/c_{\phi}) \leq e(0) \simeq 0.227$. Thus, even if periods of accelerated scale factor are allowed, they do not give rise to inflationary eras.

For a susy breaking with $n = 2$

For any fixed $n_V/n_T < 0$, the condition $\mathcal{I}(n_V/n_T, z, Z) > 0$ defines a domain in the (z, Z) -plan, as shown on Fig. 4b. Using the notations of [14], this zone spans four asymptotic sectors defined as follows,

$$\begin{aligned} II & : Z \rightarrow -\infty, z \sim \eta Z, |\eta| < \frac{1}{2}, & III & : Z \sim \eta z \rightarrow -\infty, -2 < \eta < 0, \\ IV & : Z \sim \eta z \rightarrow +\infty, 0 < \eta < 2, & V & : Z \rightarrow +\infty, z \sim \eta Z, |\eta| < \frac{1}{2}, \end{aligned} \quad (4.9)$$

inside of which $\mathcal{I} \lesssim 1$. We will evaluate the e -fold number by approximating $p(z, Z)$ by its asymptotic expansions, which are well defined in each sectors [14]:

$$\begin{aligned} p^{II} &= e^{4z} e^{-3Z} n_V S_6^o + e^{-z} e^{-Z/2} (n_T S_5^o + n_V S_5^e) + e^{3z} e^{3Z/2} (n_T + n_V) (S_4^o + S_4^e) + \dots \\ p^{III} &= e^{4z} e^{-3Z} n_V S_6^o + e^{4z} e^{2Z} n_V (S_5^o + S_5^e) + 2(n_T S_4^o + n_V S_4^e) + \dots \\ p^{IV} &= e^{4z} e^{3Z} n_V S_6^o + e^{4z} e^{-2Z} n_V (S_5^o + S_5^e) + 2(n_T S_4^o + n_V S_4^e) + \dots \\ p^V &= e^{4z} e^{3Z} n_V S_6^o + e^{-z} e^{Z/2} (n_T S_5^o + n_V S_5^e) + e^{3z} e^{-3Z/2} (n_T + n_V) (S_4^o + S_4^e) + \dots \end{aligned} \quad (4.10)$$

where the dots stand for exponentially suppressed terms and the coefficients $S_{4,5,6}^{o,e}$ are constants⁶.

⁶Their precise values are not needed in the following but can be found in the Appendix of [14].

It is interesting to note that the behavior of the temperature depends drastically on the location of the representative point of the system in the (z, Z) -plan. In sectors *III* and *IV*, one finds $\mathcal{A} \simeq 1$, $\mathcal{B} \simeq 0$ as in the 1 modulus case, while in sectors *II* (or *V*), one has $\mathcal{A} \simeq 4/3$, $\mathcal{B} \simeq 1/6$ (or $-1/6$). Using the energy-momentum conservation in the form of Eq. (3.13), this reaches

$$II : \frac{M^2 e^Z}{a^6 T^8} = \text{cst.} , \quad III \cup IV : a T = \text{cst.} , \quad V : \frac{M^2 e^{-Z}}{a^6 T^8} = \text{cst.} . \quad (4.11)$$

However, the Eqs of motion for the scalar fields are identical in all these asymptotic sectors,

$$\left\{ \begin{array}{l} \frac{1}{\frac{1}{2}(\dot{M}/M)^2 + \frac{1}{4}\dot{Z}^2 + \frac{1}{2}\dot{\phi}_\perp^2 - 3} (\ddot{M}/M)^\circ - \dot{M}/M - 4 = 0 \\ \frac{1}{\frac{1}{2}(\dot{M}/M)^2 + \frac{1}{4}\dot{Z}^2 + \frac{1}{2}\dot{\phi}_\perp^2 - 3} \ddot{Z} - \dot{Z} + 6 = 0 \\ \frac{1}{\frac{1}{2}(\dot{M}/M)^2 + \frac{1}{4}\dot{Z}^2 + \frac{1}{2}\dot{\phi}_\perp^2 - 3} \ddot{\phi}_\perp - \dot{\phi}_\perp = 0 \end{array} \right. \quad (4.12)$$

where the fractions in front of the second derivatives must have the sign of n_V *i.e.* negative.

When $\dot{M}/M + 4 \neq 0$, it is easy to show that there exist constants $c_\phi \neq 0$, c_z , c_\perp such that

$$\left\{ \begin{array}{l} 2(\dot{M}/M)^\circ = (\dot{M}/M + 4)\mathcal{P}(\dot{M}/M) \\ \dot{Z} - 6 = 2 \frac{c_z}{c_\phi} (\dot{M}/M + 4) \\ \dot{\phi}_\perp = \sqrt{2} \frac{c_\perp}{c_\phi} (\dot{M}/M + 4) \end{array} \right. \quad (4.13)$$

where

$$\begin{aligned} \mathcal{P}(x) = [1 + 2(c_z/c_\phi)^2 + 2(c_\perp/c_\phi)^2] x^2 + 4[4(c_z/c_\phi)^2 + 3c_z/c_\phi + 4(c_\perp/c_\phi)^2] x \\ + 4[8(c_z/c_\phi)^2 + 12c_z/c_\phi + 8(c_\perp/c_\phi)^2 + 3] \end{aligned} \quad \text{is of the sign of } n_V. \quad (4.14)$$

The condition (4.3) is also identical in all asymptotic sectors *II*, ..., *V*,

$$\mathcal{K} \simeq \frac{1}{2} (\dot{M}/M)^2 + [(c_z/c_\phi)(\dot{M}/M) + 3]^2 + (c_\perp/c_\phi)^2 (\dot{M}/M + 4)^2 \lesssim 1, \quad (4.15)$$

where we have used the second and third equations of (4.13). A period of accelerated cosmology exists if

$$(c_\perp/c_\phi)^2 < \frac{1}{14}, \quad c_- < c_z/c_\phi < c_+ \quad \text{where} \quad c_\pm = \frac{-6 \pm 2\sqrt{2[1 - 14(c_\perp/c_\phi)^2]}}{7}. \quad (4.16)$$

It begins and ends at most when \dot{M}/M saturates the inequality (4.15) *i.e.* equals

$$l_{b,e} = \frac{-2[4(c_z/c_\phi)^2 + 3c_z/c_\phi + 4(c_\perp/c_\phi)^2] \pm \sqrt{14(c_+ - c_z/c_\phi)(c_z/c_\phi - c_-)}}{1 + 2(c_z/c_\phi)^2 + 2(c_\perp/c_\phi)^2}. \quad (4.17)$$

To find the corresponding scale factors a_b , a_e , we integrate the first equation of (4.13),

$$\ln\left(\frac{a}{a_0}\right) \equiv \lambda - \lambda_0 = \gamma \ln|\dot{M}/M + 4| + \gamma_+ \ln|\dot{M}/M - l_+| + \gamma_- \ln|\dot{M}/M - l_-|, \quad (4.18)$$

where

$$\begin{aligned} l_\pm &= \frac{-2[4(c_z/c_\phi)^2 + 3c_z/c_\phi + 4(c_\perp/c_\phi)^2] \pm \sqrt{10(C_+ - c_z/c_\phi)(c_z/c_\phi - C_-)}}{1 + 2(c_z/c_\phi)^2 + 2(c_\perp/c_\phi)^2}, \\ \text{with } C_\pm &= \frac{-6 \pm \sqrt{7[3 - 10(c_\perp/c_\phi)^2]}}{5}, \\ \gamma &= \frac{2}{1 + 2(c_z/c_\phi)^2 + 2(c_\perp/c_\phi)^2} \frac{1}{(l_+ + 4)(l_- + 4)}, \\ \gamma_\pm &= \pm \frac{1}{1 + 2(c_z/c_\phi)^2 + 2(c_\perp/c_\phi)^2} \frac{1}{(l_\pm + 4)(l_+ - l_-)}. \end{aligned} \quad (4.19)$$

Inserting (4.17) in (4.18), the maximum e -fold number is found to be

$$e(c_z/c_\phi, c_\perp/c_\phi) = \ln \frac{a_e}{a_b} = \gamma \ln \left| \frac{l_e + 4}{l_b + 4} \right| + \gamma_+ \ln \left| \frac{l_e - l_+}{l_b - l_+} \right| + \gamma_- \ln \left| \frac{l_e - l_-}{l_b - l_-} \right|, \quad (4.20)$$

that satisfies $e(c_z/c_\phi, c_\perp/c_\phi) \leq e(-3/2, 0) \simeq 0.189$. The order of magnitude of the e -fold number for $n = 1$ and $n = 2$ is thus the same. They cannot account for the astrophysical observations.

5 Summary, limitations and perspectives

The present work focuses on an intermediate cosmological era, $t_E \leq t \leq t_w$. The time t_E corresponds to the exit of an Hagedorn phase *i.e.* after the notion of topology and geometry emerge, with 3 large space-like dimensions and possibly internal directions of intermediate scales. The characteristic time t_w is associated to the electroweak phase transition, occurring before large scale structure formations in the universe.

To describe the intermediate era, we consider flat classical 4-dimensional space-times within the context of superstring theory, where $\mathcal{N} = 8, 4, 2$ supersymmetry is spontaneously broken. Our aim is to determine the different cosmological behaviors that can be induced when three kinds of effects are taken into account. Namely,

- statistical physics effects at temperature T ,
- radiative corrections of magnitude M , the spontaneous supersymmetry breaking scale,
- the presence of moduli fields, whose kinetic energies scale as $1/a^6$.

We determine the low energy effective action at the one-loop level and parametrize data at the exit t_E of the Hagedorn phase by a set of IBC.

To be specific, when $n = 1$ internal radius is involved in the supersymmetry breaking, we show that, depending on the R-symmetry charges of the massless spectra, the models can be grouped in different *Cases* (a), (b) or (c). For each, there exists a partition of the set of IBC, whose classes yield cosmological evolutions that share a common behavior.

- In *Case* (a), one class of IBC seems to induce a spontaneous decompactification of the internal direction involved in the breaking of supersymmetry. It is out of the scope of the present paper.
- In *Case* (b), the evolutions of the scale factor are monotonic. The expanding solutions belong to the same class and are attracted to a radiation dominated era. Asymptotically, one has $3H^2 \propto 1/a^4 \propto T^4 \propto M^4$, where $a(t) \propto \sqrt{t}$.
- In *Case* (c), a class of IBC reaches solutions attracted to a Big Crunch era dominated by the radiative corrections to the initially vanishing vacuum energy. Asymptotically, one has $3H^2 \propto M^4 \propto 1/a^{24} \propto T^{24}$, where $a(t) \propto (t_{\text{BC}} - t)^{1/12}$.
- In all *Cases*, a class of IBC reaches solutions attracted to a Big Crunch era dominated by the classical moduli kinetic energy. Asymptotically, one has $3H^2 \propto 1/a^6 \propto T^6 \propto M^{1/(1-3/(8\sigma_-))}$, where σ_- is given in Eq. (2.57) and $a(t) \propto (t_{\text{BC}} - t)^{1/3}$.

Our analysis has some limitations. First of all, we have supposed that the radii involved in the spontaneous breaking of supersymmetry and the radius of the Euclidean time are large enough to avoid Hagedorn like singularities in the partition functions. Since we found Big Crunch behaviors where $T \ll M$ are formally diverging, these evolutions must be restricted to M lower than a maximum Hagedorn bound. It would be interesting to relax this constraint by considering models free of such singularities [8].

Also, we have studied the thermal and quantum corrections at the one-loop level in flat backgrounds where moduli have some kinetic energy. This means that the induced sources

P , ρ and their backreactions on the “cold” classical backgrounds we start with must be small. Thus, we need afterwards to check the consistency of our approach.

- For the RDS, we have

$$P(t) \propto \rho(t) \propto \frac{1}{a(t)^4} \propto \frac{1}{t^2} \longrightarrow 0, \quad (5.1)$$

which shows its validity.

- For the Big Crunch attractor $a(t) \propto (t_{\text{BC}} - t)^{1/12}$ of the MDS, both the thermal/quantum sources and the classical moduli kinetic energy are diverging. Formally, one has

$$P(t) = -\rho(t) \propto M^4 \propto \frac{1}{a(t)^{24}} \propto \frac{1}{(t_{\text{BC}} - t)^2} \gg \frac{1}{a^6} \propto \frac{1}{\sqrt{t_{\text{BC}} - t}}, \quad (5.2)$$

which shows that this solution has to be restricted to times where P and ρ do not exceed too much the classical kinetic energy.

- This limitation does not exist for the Big Crunch behavior $a(t) \propto (t_{\text{BC}} - t)^{1/3}$. Even if the thermal/quantum sources and the classical moduli kinetic energy are diverging, one has

$$P(t) = -\rho(t) \propto M^4 \propto \frac{1}{a(t)^{24-9/\sigma_-}} \propto \frac{1}{(t_{\text{BC}} - t)^{8-3/\sigma_-}} \ll \frac{1}{a^6} \propto \frac{1}{(t_{\text{BC}} - t)^2}, \quad (5.3)$$

since $\sigma_- < 1/2$. Thus, the backreaction on the classical background is always small. The dynamical attraction to this solution is only limited by the Hagedorn transitions mentioned previously.

To go beyond the constraint that the thermal/quantum corrections must be small, one can try to compute the higher string loop corrections and, if possible, use string-string dualities to find the non-perturbative corrections to the effective action. For the models under consideration in this work, the extended supersymmetry is only broken spontaneously, at the scale M and/or by thermal effects at the scale T . This means that the UV behavior of the theories is not affected by these soft breaking and that the renormalization theorems for extended supersymmetries should persist. To find the cosmological evolutions beyond the first order in perturbation theory, both the thermal effective potential and the corrections to the kinetic terms have to be computed.

The attraction phenomena we found can be generalized to models where n internal radii are involved in the spontaneous breaking of supersymmetry. In particular, we have shown that for $n = 2$, the attraction to the radiation era persists. In the process, the complex

structure modulus associated to the ratio of the two radii converges to a constant *i.e.* is dynamically stabilized.

During the convergence to the radiation dominated era, the ratio M/T reaches its critical value exponentially or with damping oscillations. Periods of accelerated expansions are also allowed. However, the e -fold number seems to depend weakly on n and, at least for $n = 1, 2$, does not give rise to enough periods of inflation (e -fold ≤ 0.2). Since we have considered models with $\mathcal{N} = 8, 4, 2$ initial supersymmetry (*i.e.* before spontaneous breaking), it would be interesting to extend our work to $\mathcal{N} = 1$ models and see if the order of magnitude of the e -fold is higher.

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